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AIR MINISTRY M.R.P.152.
6th December, 1943.

METEOROLOGICAL RESEARCH COMMITTEE

Report on a trial of Symmetry Point method of Long Range Forecasting carried out at the Central Forecast Station, England.

October 1942 - April 1943.

by E.G.Bilham.

METEOROLOGICAL OFFICE
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Part I. Introduction

For some years past the Climatology Branch of the Meteorological Office has included a small section for the investigation of methods of long range forecasting. In June 1940 this Section started an investigation of the claims of Weickmann with regard to waves of pressure in the atmosphere and their resultant symmetry points using current weather maps. The area for which exact current pressure values were available was relatively small and it was decided at the meeting of the Meteorological Research Committee in July 1941 to ask the section to carry out an investigation with Polar Year maps representing the most complete data available. Charts of the whole northern hemisphere were used for harmonic analysis and symmetry points were worked out for a large number of stations. On the basis of this work a number of test "forecasts" were made e.g. the weather for January 1933 was forecast from data ending on December 31, 1932. The Meteorological Research Committee decided that the resultant forecasts showed sufficient promise to warrant a full scale six months' trial using current data at the Central Forecast Station. A staff of thirty, mainly computing assistants, was allocated for the work and after two months' training in the Climatological Branch they were transferred to the Forecast Branch in October 1942.

The primary object of the trial was to produce forecast maps of isobars for three, six, nine and twelve days beyond the date on which the forecast was issued. As it took three days to complete the analyses and assess the data it was virtually equivalent to forecasting six, nine, twelve and fifteen days ahead of data used. Owing to war-time difficulties the data themselves were in many cases far from complete. The maps produced were purely isobaric - giving only the distribution of pressure - and extended from 30° to 80°N and 30°W to the Black Sea on the scale of 1:4x10 along 55°N. The secondary object was to produce general forecasts of tracks of depressions and the nature of the air masses in the neighbourhood of the British Isles.

The staff of the section consisted of one technical officer who had been working on the theory since 1940, five supervisors (graduates) who had been recruited into the Meteorological Office during the war, and twenty-four temporary assistants - some civilian, others members of the W.A.A.F.

Part II. Details of methods used and organization.

The methods used were essentially those set out by Weickmann in "Wellen im Luftmeer". He asserts that there are certain oscillations of pressure in the atmosphere which are periodic or quasi-periodic in nature. As a result of the work done by the Leipzig school it has been claimed that 72 days and the various harmonics of this interval are the periods most frequently exhibited by the oscillations. The Leipzig school is also responsible for developing the idea of the "symmetry point", while Karl Stumpff carried the idea further by means of the symmetry index,

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the ultimate object being to apply it to the problem of long range forecasting. When a symmetry point occurs on a pressure graph the curve repeats itself in reverse order from this point onwards. The similarity between the curve and its reflected image exists only in the broad features and does not generally extend to the smaller details. "Inverse symmetry", in which a low in the "reflected" half of the curve corresponds to a high in the first half, and vice versa, may also occur. Weickmann explained the existance of symmetry-points in terms of the Fourier harmonics - direct symmetry occurring when the crests of troughs of all constituent waves of significant amplitude coincide, and inverse when their null points agree. He also showed that symmetrical properties can sometimes persist for many months especially after the main winter symmetry point which occurs generally at the end of December or beginning of January.

The methods by which the tracks of depressions and polar outbreaks were forecast, although not initiated by Weickmann, had been developed by the Leipzig school. The 24-hour wave of pressure is claimed to be a purely surface phenomenon intimately associated with major outbreaks of polar air - the outbreak occurring when this wave reaches its maximum amplitude within the cycle - i.e. roughly every 24 days - while the 72 and 36-day waves extend into the stratosphere and appear to be steering waves, that is to say, when they reach their maximum amplitude in any given centre the mass of air piled there precludes the passage of depressions across it. They are in fact steered around the wave humps. Conversely halfway through the cycle when the dominant wave is at a minimum, depressions may cross through the centre of the hump without hindrance. The methods to be used in the trial thus fall naturally into two distinct parts - one involving harmonic analyses to obtain the Fourier coefficients and the other correlation of pressures to obtain an index of symmetry.

The calculation of symmetry indexes for a number of stations for each day formed the major part of the work of the computing section. Within the area for which forecast isobars were required a hundred stations were chosen to give a good network. Every day the 0100 hr. pressure was extracted for each station, overlapping 3-day sums of pressure listed and the symmetry index found. Within the area for which actual observations were not available, deduced values were used. The symmetry index is a type of correlation coefficient which evaluates the degree of correspondence between the pressures equidistant in time from a given day. $S = \frac{1}{2}(1-r)$ where r is the correlation coefficient and S is the index of symmetry - thus S = 0 indicates perfect symmetry for then r = +1.0, and S = .5 complete lack of symmetry; perfect inverse symmetry (r = -1.0) would be indicated by S = 1. When r = 0.8 two thirds of the variability of pressure on either side of the day in question is accounted for by symmetry; thus values of $S \leqslant 0.1$ would give a reasonably good symmetry day which could be used to forecast pressure. Allowance to be made for secular change is computed according to the method of least squares. If x_1 and y_1 are pressure values (expressed as departures from the mean value) at equal intervals before and after the symmetry point, then the value of S is given by the expression

$$S = \frac{\Sigma (x_1 - y_1)^2}{\Sigma (x_1 - y_1)^2 + \Sigma (x_1 + y_1)^2}$$

where x and y cover 30 days before and after the day for which the index is calculated. In practice it was assumed that a value of $S \leqslant .20$ would indicate symmetry sufficiently persistent to be of use several days after the period involved in its calculation.

It was found that symmetry days occurred regionally so that

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stations in a certain region showed symmetry on roughly the same day. When a forecast was required the latest computed symmetry day for each station was listed; then the pressure graph for the station was inspected to see if the symmetry was still holding, if so, the pressure for the next twelve days was obtained in the following manner. Suppose December 30 to be a symmetry day, then pressure for 1st February should be the same as that for the 29th November, and that for 2nd February the same as the 28th November and so on. It was found necessary to use 3-day means of pressure in order to smooth from the pressure graph any disturbing features such as abnormally deep depressions occurring before the symmetry day. The forecast pressure for each day was plotted on the chart and the isobars drawn to give the final map.

The Fourier analysis of the pressures over the Northern hemisphere was carried out by graphical methods. As a result of the German work the fixed interval of 72 days was chosen as a basis of analysis, a twenty-four ordinate scheme being used to obtain the first six harmonics. The method differed from that of Weickmann who studied the pressure graph of each separate station, obtained the phase and amplitudes of each harmonic and then plotted the distribution on a map. The L.R.F. Section adopted the method of Egersdörfer who took the pressure map for a given day as one ordinate and added, subtracted or multiplied the charts together graphically in the manner of isallobaric charts to obtain the distribution over the map of one particular wave. The formula used was the standard harmonic formula but instead of treating each station separately the map was considered as a whole. In this way a more complete picture of the wave was shown than could have been obtained by a network of 500 stations.

In addition, the pressure graphs for 80 representative stations were drawn using two observations per day, and analysed by means of the Mader and Coradi harmonic analysers to find the first six harmonics of the 72-day interval. For any particular station the phase and amplitude given by the machine should agree with that given by the charting method and this served as a check of results of the charting. The results of the machine however were more accurate for an individual station as the sine and cosine terms were correct to the nearest decimal point while from the charting the values had to be interpolated between an interval in the isopleths of 10 units. Unfortunately the machines exhibited slight defects which vitiated their readings. Hence throughout the trial the section was virtually deprived of their indications, and all the results for individual stations had ultimately to be taken from the charts by interpolation.

When the distribution of the various harmonics of the 72 day interval was ascertained, values of sine and cosine for a set number of stations were extracted from charts, checked by machine results and the phases and amplitudes computed. From these results phase diagrams for chosen stations were drawn with vertical axis for phase in degrees and a horizontal time scale. A fresh analysis of 72 days was completed every three days, i.e. moving on by overlapping three days, so every third day a new phase could be plotted on each diagram. The resulting diagrams were used to show the true length of the wave, for when the various phases given by succesive analyses for each wave for each station are joined together they should form a straight line if the wave is real, and the time taken by the wave to travel through 360° gives its true length. Hence, from the phase diagrams the validity of the set waves was estimated.

Care must be exercised in using this method to see that the corresponding section of the pressure graph is not too short. In any case the results may not be entirely free from ambiguity. When the dominant waves in operation over the map were settled

they were extrapolated for the days needed for the forecast. It was assumed that for the next fifteen days the waves would continue to develop as they had been during the previous 72 days. The extrapolation was a simple application of the formula, amplitude on any day = a cos θ + b sin θ where a and b are Fourier coefficients obtained in charting and θ = angular distance in time from the origin.

When the various waves had been added together for each of the days, 6, 9, 12 and 15 beyond the interval analysed they were added to the average pressure for the last 72 days (obtained from a_0, the constant term of the Fourier expansion) and thus maps of the forecast pressure were obtained. At the beginning of the trial the symmetry points were very unreliable - breaking down after a short time so that extrapolation alone had to be relied on to give a fair idea of the forecast pressure chart.

Later in the trial various modifications were made in the method. A shorter chain method of harmonic analysis designed by Dr. J.C.P.Miller was introduced which involved considerably less work that the previous method and was based on the principle of substituting the new ordinate for that 72 days before rather than moving each ordinate on 3 days. This involved using a fixed origin. To shorten the work still further the secular change correction was obtained from the first and last ordinates only - previously the least square method had been employed. It was also found more satisfactory to use the mean of three days pressure for each ordinate rather than the pressure map for every third day. A further innovation was the analysis of a 144 day interval to obtain the 72 day wave as a second harmonic and the 48 day as a third. This was begun after finding that the secular correction to a first harmonic was generally very large. Later on the 72 day wave seemed to have shortened to about sixty days in length and the 24 day to twenty days so the analysis of a sixty day interval was introduced. In a further effort to obtain the really dominant wave in operation the system of smoothing recommended by Vercelli was introduced. This system cut out all short waves emphasising only those between 18 and 72 days.

Early in the trial it was evident that some method of forecasting the position of symmetry points was necessary as in this season they seemed to last for about thirty days which meant they were already breaking down at the time of discovery and so were of little use. The following methods were used therefore to discover symmetry dates in advance. The last six links of the analyses for each wave were examined, the areas of significant amplitude were mapped out and the area common to all links obtained. The average date of maximum and minimum in this centre was calculated and previous symmetry days examined to see if they agreed with a previous elongation of the wave. If so it was assumed that the next symmetry day would be at the next elongation. In cases where several wave humps were in operation in the same area the date at which all the waves were next at the max or min was looked for as a symmetry day. This method gave one date as the symmetry day for a flat area. The exact day had then to be fixed for each station. This was done by means of the symmetry point slide rule. This slide rule consisted of ten moveable scales and had one line of origin drawn across the slide at the left hand and against this line the date of origin was set and also the phase of each harmonic at the date of origin obtained from machine or graphical analysis. On each of the slides the max and min and null points of the waves were marked in accordance with the time scale. When the rule was set for a station the cursor was moved along to the date suggested by the area method and the symmetry day was finally chosen as the date on which all the waves with significant amplitudes showed the closest alignment

of their crests and troughs. If required the value of the symmetry index on the day selected could be calculated. Another method of determining the symmetry point in advance was by means of phase diagrams. Each station had the phase diagrams for its six harmonics plotted on the same sheet with the same horizontal and vertical scales. To forecast the next symmetry day the phase lines were continued beyond the interval actually analysed to a point beyond the rough date set by the area method and the phases of the various waves studied. At the final day chosen as a symmetry day the phase lines of the dominant waves should be crossing either the 90° or 270° lines i.e. be at their max or minimum. It should be understood that this routine had to be repeated every three days in an attempt to settle the symmetry day more definitely, but as symmetry days occur about once every thirty days it will be seen that to find another symmetry point much repetition of work was necessary using the more up to date information available after each analysis. When the symmetry points had been predicted the forecast maps were prepared in the manner previously described - these being checked by simple extrapolation and synthesis of the dominant waves.

In addition to the forecast isobars statements about tracks of depressions and air masses were issued. These were made after examination of the latest charts of phase and amplitude of each harmonic. The Leipzig school had reached the conclusion that 36 or 72 day waves sometimes have steering properties. First of all therefore the position of the centres of max amplitude for these long waves were found on the charts - only centres with few traversing phase lines being considered to have steering properties as they are the most stable. The previous dates of max and min of such centres were calculated and their steering effect around these days checked by the tracks of isallobaric nuclei and the distribution of the 6 and 8 day wave around these dates. The 6 and 8 day waves should travel round and have amphidromes within the centre of the long wave. Thus positive centres of the 72 and 36 day waves which had been shown to steer depressions in the past were expected to show similar properties at the time of their next max and a forecast of tracks was made accordingly. Use was also made of steering symmetry - i.e. symmetry in the tracks about a given day. From previous data and its symmetrical properties the steering over the next 12 days was inferred, this being found to support the forecast of tracks of centres of depressions obtained by individual wave steering.

The air mass statements were based on the 24 day wave in its northern centre only - in the neighbourhood of the British Isles. Previous max and min of the wave centre were correlated with previous outbreaks of polar air and influx of warm air to Northern latitudes respectively. From this the next outbreak was forecast and checked by a thermal wind construction using the extrapolated 72 and 36 day waves on any day as the upper wind and 24 day wave as the lower one.

PART III. Assessment of Results.

Forecasted Pressure Charts. During the progress of the trial, copies of the forecast charts and of the notes appended thereto by the Officer in Charge of the L. R.F. Unit were passed regularly to the Forecast Branch (M.O.2(A)) for information. In view, however, of the experimental nature of the forecasts no operational use was made of them. At the conclusion of the trial, Mr. C.K.M.Douglas, the Head of M.O.2(A) prepared a Report which is appended hereto as Appendix I. It should be noted that

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Mr.Douglas's assessment was of a purely general and qualitative character, the aim being to record as fairly as possible the impressions and reactions of a synoptic meteorologist. It is well known that the major difficulty confronting a forecaster who attempts to extend the "Outlook" beyond the next day or two, is to foresee changes of type, and to time the onset of a change of type if it is foreseen. The general pattern of the isobars in the forecast chart is therefore of far more consequence than the numerical values attached to them. It is not too much to say that if Mr.Douglas had been favourably impressed by the degree of resemblance between the forecast and actual pressure pattern, the results of any purely numerical comparisons could have been set aside. It will be seen however that Mr.Douglas was unable to form any such favourable impression.

For the purpose of obtaining a quantitative estimate of the degree of success achieved in forecasting the pressure at specified places, records were kept of the actual and forecasted pressures at a number of check points. The points selected for comparison were

Vestmannö	Berlin*
Archangel	Gibraltar
Röst*	Horta
Kew	50°N 30°W
Valentia	

*"Actuals" were post facto estimates.

The values tabulated were three-day means in each case. The correlation coefficients obtained between the actual values and the values forecasted 12 days previously were as follows, for the period November 1942 to March 1943

Vestmannö	$-.13 \pm .09$	Berlin	$+.19 \pm .09$
Archangel	$+.18 \pm .09$	Gibraltar	$-.05 \pm .10$
Röst	$+.21 \pm .09$	Horta	$-.51 \pm .07$
Kew	$-.25 \pm .09$	50°N, 30°W	$-.22 \pm .09$
Valentia	$-.19 \pm .09$		

All stations $+ .176 \pm .03$

It will be seen that there is no significant correlation between the actual and forecast pressures.

As a further test, correlation coefficients were evaluated between the forecasted values of the pressure gradients between Horta and Vestmannö, Valentia and Berlin, and Vestmannö and Röst, and the actual values. The values obtained were

Horta - Vestmannö	$-.115 \pm .096$
Valentia - Berlin	$-.029 \pm .094$
Vestmannö - Röst	$-.046 \pm .094$

This test might be expected to give a definte indication of the value of the method as a means of forecasting the general isobaric pattern. These results, however, show that during the period under review the forecast charts were of no value as a guide to the future pressure field.

Air Mass Forecasts.

In the early part of the trial these inferences were not successful. The degree of success improved later, but it is not certain that this may not have been an accident arising from the type of weather experienced.

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Forecasts of Tracks of Depressions.

During the period of the trial, forecasts of the tracks of depressions were made on the principles explained in Part II. A statistical analysis of these forecasts was not carried out. A subsequent analysis of the 36-day wave of pressure during 1942-43 showed that this pressure wave did not appear as a quasi-stationary oscillation possessing steering properties, in a manner postulated by the Leipzig school, except during September and October 1942, that is to say, prior to the commencement of the trial period.

General Conclusions.

The result generally is disappointing. It is obvious, of course, that the utility of the symmetry method must depend upon (a) the precision with which symmetry dates can be identified or predicted, and (b) the degree of resemblance between the pressure graph to the symmetry date and its reflected image. As regards (a), a post facto investigation of the errors made in fixing symmetry dates gave the following distributions.

Error in date (days)	0	1	2	3	4	5	6	7	8
Number of cases	36	56	33	32	15	15	13	20	6

The number of errors exceeding 8 days was relatively small. The mean error was approximately 3 days.

As regards (b), it was estimated that if there had been no errors at all in the forecasts of symmetry dates, the correlation coefficient between the actual and forecast pressures would have been +.49± .03. A correlation coefficient of this order of magnitude is of little value for forecasting, as it means that about 87 per cent of the variations of pressure are due to factors other than symmetry.

On the other hand, it must be remembered that the correlation coefficient affords no means of allowing for slight displacements in timing of events, and that high values were hardly to be expected even in cases where the symmetry is deemed to be good by other standards of assessment. Generally speaking, the symmetry is seen only in the broader features of a pressure graph extending over longer intervals than 12 days, not in the smaller details.

Hence in addition to the risk of making errors of prediction of a symmetry point there are the errors arising from the lack of symmetry in the details of the pressure graph, and which it appears render the method unsuitable for forecasting charts of pressure distribution for 12 days ahead.

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Appendix I - Report on Results.

by C.K.M.Douglas.

Attached hereto is a comparison between the general type of weather and that predicted by the L.R.F.Unit.

The opinion formed by the Senior Forecasters and myself is that the standard is not sufficiently high to enable them to be used in the work of M.O.2(A). Our "period" forecasts are based mainly on the persistency of the existing type. In the past winter forecasts based on persistence gave apparently better results than the long-period forecast isobars. The past winter may have been abnormal in that respect. The SW type was very prolonged in January and February and also lasted for a fortnight in December. The November and March anticyclones were also unusually persistent.

It is impossible to judge from a general assessment as to whether the results are better than could be obtained at random.

Note on the General Type of Weather and the Type Predicted

OCTOBER

About October 21st, when the first forecast was issued, there was a SW current over the British Isles, with an anticyclone over the Bay of Biscay and low pressure in the Iceland area. Subsequently low pressure spread over the British Isles and there was a notably wet spell from 25th to 30th with depressions at times centred over the British Isles. Wind became NE to N from 28th to 31st.

The forecast showed unsettled westerlies throughout this period, this type being maintained on the forecast issued on October 27 for 29th.

NOVEMBER

The situation over the British Isles was flat for the first five days of November, and became southwesterly on 6th with a very deep depression over Iceland. An anticyclone over theBay of Biscay on 8th moved NE to the southern North Sea next day and began a long anticyclonic period. A new anticyclone came in from the Atlantic and assumed marked intensity over Iceland on 16th. It persisted with varying intensity till the end of the month.

The forecast issued on October 27th indicated southwesterlies on November 1st and 4th and an anticyclone over the North Sea on November 7th, but this last feature was dropped on later issues. The development of the anticyclone was not forecast beforehand though the forecast for November 19th issued on 14th shows it in substantially the right position. The forecasts for later dates

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indicated a change to a westerly type which did not materialize.

DECEMBER

A flat pressure distribution with rather cold weather changed to a mild unsettled SW type on 4th which lasted till 22nd. The forecast for December 1st showed a pronounced westerly type. The forecast issued on December 2nd showed a complex type of situation which would have given low temperature till 7th, becoming milder later but changing to a W to NW rather than a SW type.

On December 22nd an anticyclone started to spread up from south and from 25th to 27th the south of England was in the anticyclone with considerable fog.

The forecast issued on December 19th for December 22nd shows the anticyclone already over Southern England. Though this was premature, it indicated the correct change of type.

A cold northerly current became established over the British Isles on 29th. This was not indicated on the forecast charts till January 3rd.

JANUARY - FEBRUARY

A depression crossed the British Isles on 1st and in its rear there was a brief renewal of cold northerlies. The forecast for January 3rd was thus successful.

There were complex conditions with intermittent frost till January 9th, and thereafter a mild stormy SW type which continued till February 18th in the South and till March 2nd in the North. There were no major interruptions of this type, but there was a quieter interval from January 21st - 23rd and a brief cold outbreak on February 7th. The change to a mild type was not shown on the forecast issued on December 31st or January 6th, the latter indicating a complex pressure distribution till 18th.

The forecast charts showed an anticyclone over England on January 24th which was maintained till February 11th on all the issues. This had no resemblance to the actual stormy SW type which had its climax in the exceptionally severe gale of January 31st.

An anticyclone actually did cover Southern England on February 18th and intermittently till the end of the month. The forecasts showed an unsettled southwest type during this period.

MARCH

Early in the month the anticyclone spread north and conditions were generally anticyclonic till 8th. The forecast issued on February 24th shows the anticyclone covering the south on 4th and the British Isles on 7th, the indicated change being correct but late as regards timing.

Between March 8th and 17th the ridge of high pressure was either over or just south of England and rainfall was low. In north Scotland and over the area to northwards conditions were stormy with gales at times. The forecast issued on February 25th for March 10th (i.e. mean for 9-11) was substantially right for the south but it did not indicate the storminess in the North, though it gave the correct direction for the isobars.

On March 17th the type changed and was E to SE till 25th.

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The forecast for March 19th and 22nd (Nos.37 and 38) did not give the right type. No.39 shows an indefinite col over England on March 22, and a SW type on 25th. Actually winds were still east on 25th but rain had spread up and conditions were changing. The type was westerly from 26th onwards becoming stormy on 29th-31st. The forecasts were quite good but the direction of the isobars was 45° out over England and the rather low pressure and cyclonic curvature of the forecast charts would have given substantially more rain than actually fell.

APRIL (1st - 15th).

An anticyclone centred to SW on 1st moved east and covered England on 3rd and 4th. On 5th a depression spread from north and there was a pronounced NW type till 9th with gales on 5th in the south.

The type then became W to SW but was feebly developed except in the extreme north. Conditions were dry in the south.

The forecast showed an anticyclone over the Bay of Biscay on 3rd, south of its actual position. On 6th it showed a SW type with very low pressure to NW. The forecast for 9th and 12th showed a SW type with cyclonic curvature over the British Isles.